

14th Benelux Mathematical Olympiad

Leuven, April 29–May 1



The problems are not ordered by estimated difficulty.

Problem 1. Let $n \geq 0$ be an integer and let a_0, a_1, \dots, a_n be real numbers. Show that there exists $k \in \{0, 1, 2, \dots, n\}$ such that

$$a_0 + a_1x + a_2x^2 + \dots + a_nx^n \leq a_0 + a_1 + \dots + a_k$$

for all real numbers $x \in [0, 1]$.

Problem 2. Let n be a positive integer. There are n ants walking along a line at constant nonzero speeds. Different ants need not walk at the same speed or walk in the same direction. Whenever two or more ants collide, all the ants involved in this collision instantly change directions. (Different ants need not be moving in opposite directions when they collide, since a faster ant may catch up with a slower one that is moving in the same direction.) The ants keep walking indefinitely.

Assuming that the total number of collisions is finite, determine the largest possible number of collisions in terms of n .

Problem 3. Let ABC be a scalene acute triangle. Let B_1 be the point on ray $[AC$ such that $|AB_1| = |BB_1|$. Let C_1 be the point on ray $[AB$ such that $|AC_1| = |CC_1|$. Let B_2 and C_2 be the points on line BC such that $|AB_2| = |CB_2|$ and $|BC_2| = |AC_2|$. Show that B_1, C_1, B_2, C_2 are concyclic.

Problem 4. A subset A of the natural numbers $\mathbb{N} = \{0, 1, 2, \dots\}$ is called *good* if every integer $n > 0$ has at most one prime divisor p such that $n - p \in A$.

- (a) Show that the set $S = \{0, 1, 4, 9, \dots\}$ of perfect squares is good.
 - (b) Find an infinite good set disjoint from S .
- (Two sets are *disjoint* if they have no common elements.)

Language: English

*Time available: 4 hours and 30 minutes
Each problem is worth 7 points*